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Time Varying Cyclical Analysis for Economies in Transition

Warsaw, January 2007
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Publication was financed by Rabobank Polska SA

Keywords: Time-Frequency Analysis, Coherence, Growth Rates, Business Cycle
JEL Classification: C22, C29, C49, F43, O49

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Graphic Design: Agnieszka Natalia Bury

ISSN 1506-1701, ISBN 978-83-7178-418-7 EAN 9788371784187

Publisher:
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Abstract

The identification of a possible European business cycle has been inconclusive and is complicated by the enlargement to the new member states and their transition to market economies. This paper shows how to decompose a business cycle into a time-frequency framework in a way that allows us to accommodate structural breaks and nonstationary variables. To illustrate, calculations of the growth rate spectrum and coherences for the Hungarian, Polish, German and French economies show the instability of the transition period. However, since then there has been convergence on the Eurozone economy at short cycle lengths, but little convergence in long cycles. We argue that this shows evidence of nominal convergence, but little real convergence. The Maastricht criteria for membership of the Euro therefore need to be adapted to test for real convergence.
Introduction

Although the Eastern European member states joined the EU club only two years ago, there is already discussion about when they will be able to join the Euro. Some see a tight exchange rate band within the groups of candidates as a necessary pre-condition (Sell, 2000). Others, the Kok report (2004) for instance, argue that the candidates’ fiscal expansions need to be better controlled. In a similar vein, Borowski (2004) argues that on balance Poland would benefit significantly from EMU membership while Mandel and Tomsik (2003) highlight the importance of real as well as nominal convergence. Janacek and Janackova (2004) note that the new member states will not form an optimal currency area. So the catch up process is the relevant issue – not Euro membership. The key question is: how far have the new member states converged to Eurozone performance?

This question is not an easy one to answer. From a theoretical perspective, neoclassical growth models show that every economy approaches a steady-state income level determined by the discount rate, the elasticity of factor substitution, the depreciation rate, capital share, and population growth. Once at the steady-state, the economy grows at a constant rate. Thus, to the extent that the determinants of the steady-state are similar across economies, convergence is expected. But if these determinants are different, they will not converge. For example Sala-i-Martin (1996) shows a wide convergence of regions within countries. Mankiw et al. (1992) also find evidence of convergence for a sample of OECD countries at similar level of development over the years 1960-1985. But they reject the convergence hypothesis in a wider sample of 75 economies when structures and the degree of uncertainty vary a good deal more, as they do in the new accession countries. Dowrick and Nguyen (1989), Wolff (1991), Barro and Sala-i-Martin (1992), Quah (1993) all reach similar conclusions.

As far as the Eurozone is concerned, Artis and Zhang (1997), and Frankel and Rose (1998) argue that if exchange rates are successfully pegged, business cycles are likely to converge. On the other hand, Inklaar and de Haan (2000) do not find any evidence for a European business cycle in practice; while Frankel and Rose (1998) and Prasad (1999) argue that growing trade and financial links should lead to more correlation between business cycles – especially within EMU. In fact Chauvet and Potter (2001) find that the US business cycle was in line with the G7 from the mid 70s, but then diverged thereafter. Kiani and Bidarkota (2003), Stock and Watson (2002, 2003) likewise find divergence caused by structural breaks. All these results suggest a time-varying approach is going to be necessary if we are to analyse the state of convergence among economies that are in transition from
planned to market economies. But structural characteristics will be important too. It appears that cyclical correlations typically fall with the degree of industrial specialisation which increases, both in Europe and beyond, as trade and financial integration intensify. In fact, most recent studies suggest little convergence has taken place, and possibly divergence driven by increasingly specialised industrial structures (Doyle and Faust, 2003; Kalemli-Ozcan et al., 2003; Peersman and Smets, 2005).

The studies cited above all stress that the results in this literature are sensitive to: a) the choice of coherence measure; b) the choice of cyclical measure; and c) the choice of detrending measure used to identify the cycles. These sensitivities are emphasised in Canova (1998). The advantages of a time-frequency approach are therefore:

- It does not depend on any particular detrending technique, so we are free of the lack of robustness found in most of the recent studies.
- Our methods also do not have an “end-point problem” – no future information is used, implied or required as in band-pass or trend projection methods.
- There is no arbitrary selection of a smoothing parameter, such as in the HP algorithm, or (equivalently) any arbitrary band-pass selection (Artis et al., 2004).

Nevertheless, any spectral approach is going to depend on a weighted sum of sine and cosine functions. This is not restrictive if the spectra are allowed to be time-varying. Any periodic function may be approximated by its Fourier expansion – a possibility that encompasses non-differentiable functions, discontinuities and step functions. Hence, once we have time-varying weights (the time-frequency approach) we can get almost any cyclical shape. For example, to get long expansions, but short recessions, we need only a regular business cycle plus a longer cycle whose weight increases above trend but decreases below trend. This is important because many observers have commented on the typical shape of economic cycles: long expansions, short recessions; expanding cycle lengths; and steeper up phases than down phases (Peersman and Smets, 2005; Stock and Watson, 2002). A time-varying spectral approach is therefore necessary to provide the flexibility to capture those features; and the possibility of structural breaks in a period that covers the transition to market economies, a transition to the European single market, the advent of the Euro, and enlargement.
1. Time-Frequency Estimation: a guided tour

The Fourier transform (FT) is the standard tool for spectral analysis in the area of signal processing. However, the FT is inadequate when the signal is nonstationary. Classical Fourier techniques only reveal the overall frequency content of these signals. Often it is more important to know when those frequency components are present, and how they change.

As the conventional representations in the time domain or the frequency domain are inadequate in the situations described above, an obvious solution is to seek a representation of the signal as a two-variable function whose domain is the two-dimensional $(t, f)$ space. Its constant $t$-cross section should show the frequency or frequencies present at time $t$, and its constant $f$-cross section the time or times at which frequency $f$ is important. Such a representation is a **time-frequency representation** (TFR).

This variation may be described by a function $f_i(t)$, called the **instantaneous frequency** (IF). A signal may have more than one IF. The IF of a signal $x(t)$ is defined as

$$f_i(t) = \frac{1}{2\pi} \frac{\partial \phi(t)}{\partial t} \quad (2.1)$$

For the purposes of illustration we only consider **mono-component** signals here. A mono-component signal is described in the $(t, f)$ domain by one single “ridge”, corresponding to one region of “energy concentration”. Furthermore, interpreting the crest of that ridge as a graph of frequency against time, we require the IF of a mono-component signal to be a single valued function of time. Such a mono-component signal the has the form

$$x(t) = A(t)e^{i\phi(t)} \quad (2.2)$$

where $\phi(t)$ is differentiable and called the **instantaneous phase of the signal**, and $A(t)$ is real and positive - being the instantaneous amplitude. If $s(t)$ itself is real, it can be expressed as

$$s(t) = A(t)\cos \phi(t) \quad (2.3)$$

We also require $s(t)$ to be analytic. A signal $z(t)$ is said to be analytic if and only if

$$Z(f) = 0 \text{ for } f<0 \quad (2.4)$$
where $Z(f)$ is the Fourier transform of $z(t)$. An analytic signal therefore contains no negative frequencies, but it may have a spectral component at zero frequency.

Next we need to highlight the similarity and differences of the IF to “traditional” spectral analysis. First, we may assume a constant-frequency signal such as

$$x(t) = a(t) \cos(2\pi f_c t + \varphi) \tag{2.5}$$

where $f_c$ is the constant frequency and $\varphi$ is also constant. As $t$ increases by an increment $1/f_c$, the argument of the cosine function increases by $2\pi$ and the signal passes through one cycle. So the period of the signal is $1/f_c$, and the frequency (reciprocal of the period) is $f_c$. In view of (2.3), we now define

$$\varphi(t) = 2\pi f_c t + \varphi \tag{2.6}$$

Differentiating (2.6) with respect to $t$ and solving it for $f_c$, we obtain:

$$f_c = \frac{1}{2\pi} \frac{\partial \varphi(t)}{\partial t} \tag{2.7}$$

Although the left-hand side of this equation (the frequency) is normally assumed to be constant, the right hand side would be variable if $\partial \varphi(t)/\partial t$ was a non-linear function of time. Nevertheless, one can see the similarity to (2.1). Hence we can consider a Fourier transform of $\varphi(t)$ as a generator of time varying frequencies:

$$z(t) = a(t) e^{j\varphi(t)} \tag{2.8}$$

where $a(t) > 0$ and $j = \sqrt{-1}$. Let $\varphi(t)$ be evaluated at $t = t_1$ and $t = t_2$, where $t_2 > t_1$. If $\varphi(t)$ is differentiable, there exists an interval of time $t$, between $t_1$ and $t_2$, such that

$$\varphi(t_2) - \varphi(t_1) = (t_2 - t_1) \frac{\partial \varphi(t)}{\partial t} \tag{2.9}$$

Let $p_i$ be the period of one particular oscillation of $z(t)$, and let $f_i = 1/p_i$. If $t_2 = t_1 + p_i$ then $\varphi(t_2) = \varphi(t_1) + 2\pi$ so that equations (2.9) and (2.1) become:

$$2\pi = p_i \frac{\partial \varphi(t)}{\partial t} \quad \text{with} \quad f_i(t) = \frac{1}{2\pi} \frac{\partial \varphi(t)}{\partial t} \tag{2.10}$$
Hence, $t$ is an instant during a cycle of oscillation and $f_i$ is the frequency of that oscillation, suggesting that $f_i(t)$ is the instantaneous frequency at time $t$ as in the definition (2.1). Hence, the IF is a time-frequency representation (TFR) in which time and frequency information is displayed jointly. For practical purposes, the next question is how best to estimate the IF?

2. The Spectral Estimation of Non-Stationary Processes

In general, there are two categories of TFR estimators available (Berry et al., 2001): **Parametric** and **Non-Parametric** estimators. Parametric estimators require a priori knowledge about an underlying model or model system which can reproduce the original time series data. A popular model, and the one used in this paper as well, is the AR(p) model. The AR model consists of a system driven by zero mean white noise, with the current output depending on $p$ past scaled outputs and the current input variable. Based upon $p$ scalar values, the IF can easily be computed if the AR(p) model is time-varying. The resulting spectral estimators have been called “high-resolution” (Berry et al., 2001), because of their ability to resolve closely spaced spectral components.

The second category (non-parametric estimators) do not presume that time series data can be fitted into a specific model. The **Short-Time Fourier Transform** (STFT) is used instead (Tomazic, 1996). The idea of the STFT is to perform a Fourier transform on a block by block (a rolling window) basis, rather than to process the entire signal at once. The result of such a transform can then be thought of as a signal’s frequency behaviour during the time period corresponding to the current data block. For discrete time, and in order to highlight the similarity to the parametric estimator, the STFT $X(t, \omega)$ of a signal $x(t)$ at time $t$ and frequency $\omega$ can be written as (Narasimhan and Pavanalatha, 2004):

$$X(t, \omega) = \sum_{k=0}^{N-1} h(k-n) x(k) e^{-j\omega k}$$  \hspace{1cm} (3.1)

where $h(.)$ is a window function centred about zero. The STFT gives the spectral information of the signal within the window at its current position. By sliding the window $h(n)$ to different positions it is possible to get the time-varying spectral characteristics of the signal. However, the STFT assumes that the signal is stationary within the length of the window, where the length of the window can be equal to one. To get better results in terms of isolating the signal characteristics in time, and to reduce the non-stationarity problem, the window length would...
be reduced. But this results in a poor frequency resolution, if estimated directly. Therefore there is a trade off between frequency and time resolution. The main advantage of the STFT is its ease of implementation. It is the most efficient method for computation (Lin, 1997).

Returning to the parametric methods, all of the parametric spectrum estimation techniques, such as AR and ARMA models can be used for time-frequency analysis, if short-time or local stationarity is assumed (see Boashash and Reilly 1992, or Kay 1989). Here it is simply assumed that a signal \( s(t) \) can be generated using an ARMA process with a random time series \( \varepsilon(t) \). Mathematically that implies:

\[
s(t) = \sum_{k=0}^{p} a_k s(t-k) + \sum_{k=1}^{q} c_k \varepsilon(t-k) + \varepsilon(t) \quad (3.2)
\]

where \( a_k \ (k=1,2,\ldots,p) \) and \( c_k \ (k=1,2,\ldots,q) \) are parameters to be estimated. The power spectrum of the signal \( s(t) \) can be calculated from these parameters. By construction a time-varying adaptive ARMA filter of the output signal \( y(t) \) can be made to approximate the input signal \( x(t) \) as follows:

\[
y(t) = \sum_{k=0}^{p} a(t)_k x(t-k) + \sum_{k=1}^{q} c(t)_k e(t-k) \quad (3.3)
\]

where \( a(t)_k \) and \( c(t)_k \) are time-varying parameters and \( e(t) \) is the estimation error. By using the Fourier transform as in (2.8)-(2.10), we can derive the TFR of the spectrum:

\[
s(t, \omega) = \frac{\sigma^2}{\left| 1 + \sum_{k=0}^{q} c(t)_k e^{-j\omega k} \right|^2} \quad (3.4)
\]

where \( \sigma^2 \) is the residual variance. Thus, at any point in a time series, a power spectrum can be calculated directly from the updated parameters of the model. Similarly, the power spectrum for any particular time interval can be calculated by averaging the filter parameters over that time interval. Hence, if updated at each point of time, (3.4) can be interpreted as an STFT for each frequency with a window of 1, but avoiding the problems of cross-terms. The advantage of this method is the increased spectral and temporal resolution. Monte Carlo simulations have shown that this method provides narrow frequency peaks, permitting more precise frequency identification and an enhanced ability to determine frequency changes at any point in time (Lin and Chen, 1995). The main disadvantage is that the amplitude of
spectral peaks is largely dependent on the accuracy of the time series modelling of the signal process.

Finally, given a time-varying spectral model, one can use the coefficients to calculate the coherence or phase shift between them at any point of time. Moreover, the adaptive spectrum method is independent of the particular time series estimator chosen. The Kalman filter could be used just as well as recursive least squares for example. We in fact use the Kalman filter here.

3. Have the New Member States converged on the Euro?

By way of illustration, figure 1 shows the spectrum for output growth in Hungary – one of the accession countries for which we have reliable data from the start of the transition period in 1989.

The Hungarian spectrum is typical of many small transition economies. It is calculated from an AR(5) model using the methods described in section 3. In this case middle length cycles (\(\omega = 1.0\) to 1.5) are as important as the long cycles, except during the transition period itself 1989-96, and they become more important after 1997. The picture is dominated by the instability and general increase in variability during the transition phase. Initially this involves a collapse of the existing cycles in 1988; followed by a sharp increase in the variability of output at long cycles; and then a period of decline in the importance of the long cycles, in favour of the continued importance of medium length cycles and a moderate increase in the importance of very short cycles. This suggests some nominal short run (high frequency) convergence in which the price mechanism adjusts to Hungary’s chief trading partners. That would produce a similar pattern of cyclical importance as observed in the EU, at the expense of real convergence at longer frequencies which would represent globalisation within the world economy and give a similar cyclical pattern at the longer frequencies to the US (for example).
Coherence: We have calculated coherences for the Hungarian spectrum with those for the Eurozone and Germany separately (the latter to help uncover any evidence of “clustering” within the EU; and to make the distinction between convergence and globalisation more clear). There is strong coherence between Hungary and the Eurozone – but only at the shorter cycle lengths ($\omega \geq 1.6$) and only after 1991. Figure 2 has the details. Consequently, there is evidence of convergence after the transition period starts; but it is only at the high frequency end of the spectrum, and only with the Eurozone average – not necessarily with any specific economy.
Second, it is clear that the coherence at the short end of the spectrum ($\omega = 1.6, 2.4, 3.1$) is strong, at 70% compared to 40% or less at the long frequencies. However, there is a slight strengthening coherence at the long end by 2003. This suggests some real convergence with the Eurozone has now started to take place without losses in coherence at the short frequencies.

The big change however, is in the extreme volatility of the transition period (1992-3). Here the coherence with the Eurozone and Germany jumps (across the board) from 40% or 50% to around 90%, before settling down to 40% and 60% after the transition. That underlines the nominal nature of convergence so far. Interestingly, the coherence with Germany (not shown here) is a little higher than with the Eurozone – and it is larger at short cycles than it is at long cycles. The implication therefore is that the Hungarian economy is becoming more like a core Eurozone economy in terms of market responses and trade, but not in real terms (incomes, growth, output cycles and employment).

The Polish spectrum shows like the Hungarian spectrum a large degree of volatility in particular around 1990. It is calculated from an AR(4) model. Over the sample period the long run cycle disappeared in favour of a more common business cycle at a frequency of 1.0. Interestingly, the impact of short cycles remained constant over the sample period. However,
given that the long cycle disappeared, short cycles gained in relative weight. The collapse of the communist system therefore led to a less rigid business cycle. For convergence to happen, the Polish economy needs higher growth rates than the rest of the Eurozone. A prerequisite of higher growth rates may be that the economy is more flexible than the other countries. It seems that this is what the spectrum shows. Since the long cycle lost importance, the Polish economy became more flexible.

**Figure 3: The Polish Spectrum**

![The Polish Spectrum](image)

**Coherence:** Since the collapse of the communist system, the coherence with the Eurozone is quite high at about 70% for shorter cycles. That is in the short-run, the Polish economy really depends on the Eurozone. In the long-run though, there is scope for other influences, e.g. the US. Moreover, in comparison with Hungary, the coherence shows that the link of Poland to the Eurozone was very weak indeed during communist times: The coherence is equal to zero at all frequencies. However, most recently, the coherence of longer cycles increased to almost 68%.

In terms of convergence, the short-run high coherence between Poland and the Eurozone is important. However, long-run developments matter as well, especially with regards to monetary policy. If Poland was to join the Eurozone today, Poland can expect in the long-run asymmetric effects from the common monetary policy. The question is whether this is in Poland’s interest.
Germany. We now compare the transition economies with an established member of the Eurozone, namely Germany. The German growth rate was estimated as an AR(7) model. Before reunification, the spectrum is fairly stable with a peak at a frequency of about 1.5. After reunification Germany experienced a larger volatility as well. The long run cycle lost temporarily its importance. Over time though a new shorter cycle emerged at a frequency of 2.2. That shows that the German business cycle has become more volatile in recent years. In any case, the German economy is far from being in a steady state situation. That has implications for Germany as well as for the Eurozone. For Germany it implies that the current common monetary policy is not appropriate at the moment. For the Eurozone, it implies that one important member state is out of step with the rest. Hence, the common monetary policy may not be appropriate for the other members either.
France. We now compare the French spectrum with the German spectrum. For the French growth rate we estimated an AR(4) model. In difference to the German spectrum, the French spectrum looks very stable throughout the sample period. There is a single peak at a frequency of 2. Obviously, there are changes of the spectral density when a shock occurs (like in 1993). They key difference though is that the system returns after some time to the initial shape. Having said that, in 2003 there is a significant change of the shape of the spectrum. Since 2003, we have not yet enough observations to know what shape it will be, but the dominant cycle lost power. This may be the start of a convergence of the German and French cycle or not. In either case, all countries have their own characteristics. That raises the question, to what the new member states should converge to? One particular country of the existing Eurozone or the average?
Conclusion

These results show that a time-varying spectral analysis can be used to uncover the cyclical properties of nonstationary time series, even in extreme cases of a transition between regimes. In the case of the new member states in Europe, it shows a difference between real and nominal convergence. That suggests that the Maastricht criteria for membership of the Euro-zone would be well advised to put at least as much emphasis on ensuring real convergence as nominal convergence. It also suggests that full convergence is likely to take 20 years or more.

We could also show, that Germany undergoes a structural change at the moment as does France. In that sense Germany and the new member state do have something in common. However, it highlights that the Eurozone has not yet settled to a common business cycle. The question is whether the new member states really want to join into such a fragile situation.
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